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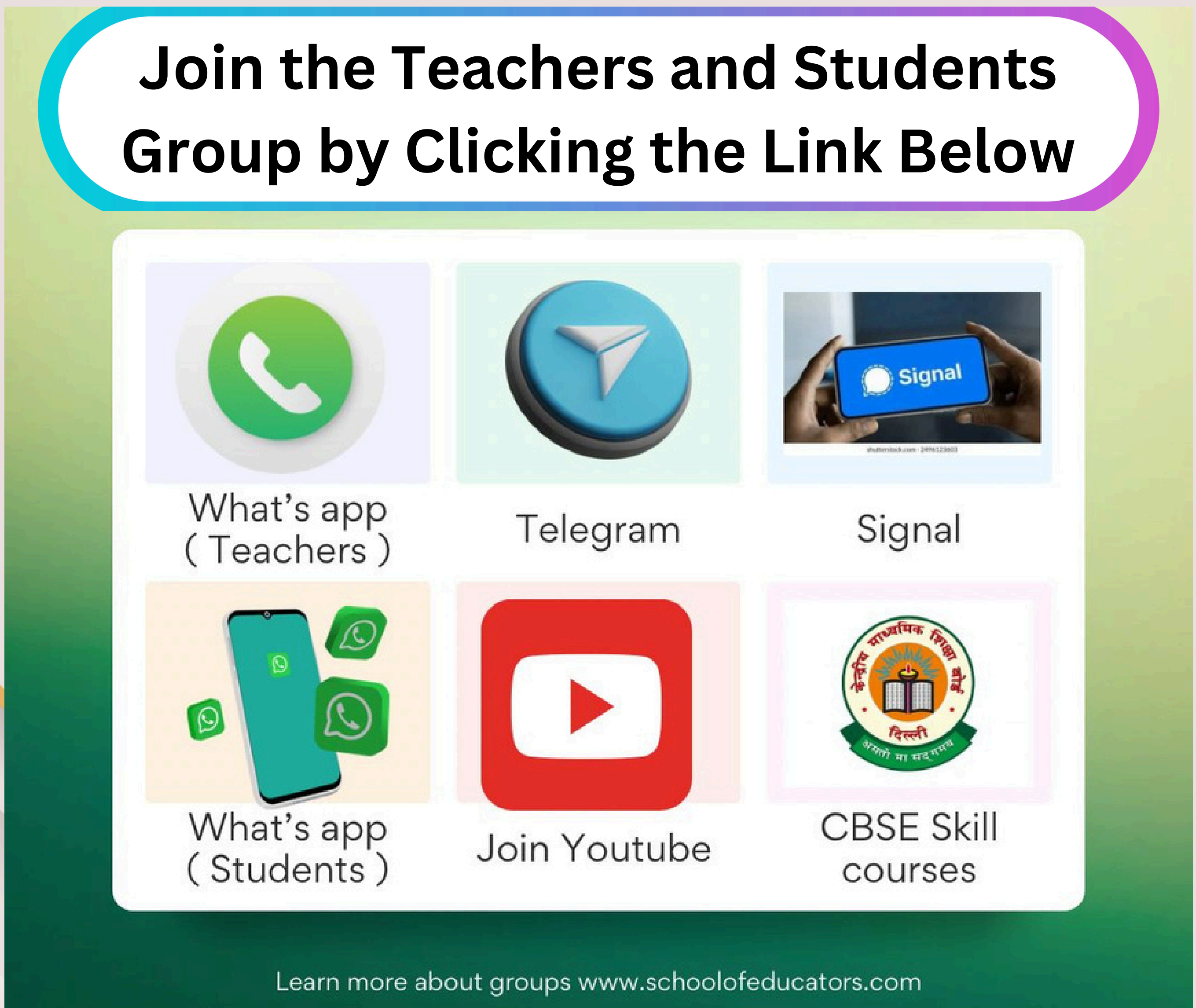


Class – 12 Mathematics

Previous Year Questions

Chapter – 2

Inverse Trigonometric Functions

INVERSE TRIGONOMETRIC FUNCTIONS

Objective Qs (1 mark)

1. The value of $\sin\left[sin^{-1}\left(\frac{1}{2}\right) + cos^{-1}\left(\frac{1}{2}\right)\right]$ is:

(a) 1
(b) 2
(c) $\frac{1}{2}$
(d) $\frac{1}{4}$

[Delhi Gov. SQP 2022]

2. The value of $sin^{-1}\left(cos\frac{13}{2}\right)$ 13? 5 is:

(a) $-\frac{3\pi}{5}$
(b) $-\frac{\pi}{10}$
(c) $\frac{3\pi}{10}$
(d) $\frac{\pi}{10}$

[CBSE Term-1 2021]

3. The value of the expression $\sec^{-1}(2) + sin^{-1}\left(\frac{1}{2}\right) + tan^{-1}(-\sqrt{3})$ is:

(a) $\frac{5\pi}{3}$
(b) $\frac{\pi}{3}$
(c) $\frac{-\pi}{3}$
(d) $\frac{\pi}{6}$

[Delhi Gov. Term-1 2021]

4 .If $a \le 2\sin^{-1} x + \cos^{-1} a \le b$, then:

(a) $a = 0, b = \pi$
(b) $a = \pi, b = 2\pi$
(c) $a = -\frac{\pi}{2}, b = \frac{\pi}{2}$
(d) $a = 0, b = \frac{\pi}{2}$

[Delhi Gov. Term-1 SQP 2021]

5. If $\tan^{-1} x = y$, then:

(a) $-1 < y < 1$
(b) $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
(c) $-\frac{\pi}{2} < y \le \frac{\pi}{2}$
(d) $y \in \left\{\frac{-\pi}{2}, \frac{\pi}{2}\right\}$

[CBSE Term-1 SQP 2021]

6. If inverse of matrix $\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ is the matrix $\begin{bmatrix} 1 & 3 & 3 \\ 1 & \lambda & 3 \\ 1 & 3 & 4 \end{bmatrix}$, then value of λ is:

(a) -4
(b) 1
(c) 3
(d) 4

(2024)

7. If $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$, then $\cos^{-1} x + \cos^{-1} y$ is:

(a) $\frac{-\pi}{3}$

(b) $\frac{\pi}{3}$

(c) π

(d) $\frac{\pi}{2}$

[Delhi Gov. Term - 1 SQP 2021]

8. The principal value of tan−1 3 − cot−1 (− 3) is:

(a) π

(b) $\frac{\pi}{2}$

(c) 0

(d) $2\sqrt{3}$

[CBSE Term-1 2021]

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R).

Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (?).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true.

9. Assertion (A): Maximum value of $\cos^{-1}$ is π^2

Reason (R): Range of the principal value branch of $\cos^{-1} x$ is $\left[\frac{\pi}{2}, \frac{\pi}{2}\right]$. .

[CBSE 2023]

10. Assertion (A): The domain of the function $\sec^{-1} 2x$ is $\left(-\infty, -\frac{1}{2}\right] \cup \left[\frac{1}{2} \infty\right)$.

Reason (R): $\sec^{-1}(-2) =-\frac{\pi}{4}$

[CBSE SQP 2022]

11. Assertion (A): $\sin^{-1}\left[sin\left(\frac{2\pi}{3}\right)\right] = \frac{2\pi}{3}$

Reason (R): $\sin^{-1}(\sin\theta) = \theta$ if $\theta \in \left[\left(\frac{-\pi}{2}\right), \frac{\pi}{2}\right]$

[Delhi Gov. SQP 2022]

12. Assertion (A) : Domain of $y = \cos^{-1}(x)$ is $[-1,1]$.

Reason (R) : The range of the principle value branch of $y = \cos^{-1}(x)$ is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$.

Very Short & Short Qs (1 - 3 marks

13. Find the value of $\sin^{-1}\left(cos\left(\frac{33\pi}{5}\right)\right)$

[CBSE SQP 2023]

14. Find the domain of $\sin^{-1}(x^2 - 4)$.

[CBSE SQP 2023]

15. Evaluate $\sin^{-1}\left(sin\frac{3\pi}{4}\right) + cos^{-1}(cos\,\pi) + tan^{-1}(1)$

[CBSE 2023]

16. Draw the graph of $\cos^{-1} x$, where $x \in [-1,0]$. Also, write its range.

[CBSE 2023]

17. Find the value of $\sin^{-1}\left[sin\left(\frac{13\pi}{7}\right)\right]$.

18. Prove that:

$$\sin^{-1}\left(2x\sqrt{1-x^2}\right) = 2cos^{-1}x, \frac{1}{\sqrt{2}} \leq x \leq 1$$

[CBSE 2020]

19. Prove that $3\sin^{-1}x = \sin^{-1}(3x - 4x^2)$, $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

[CBSE 2018]

20. Find the value of $\tan^{-1}\sqrt{3} - sec^{-1}(-2)$.

[CBSE 2018]

21. Prove that: $3\cos^{-1}x = \cos^{-1}(4x^3 - 3x)$, $x \in \left[\frac{1}{2}, 1\right]$

[CBSE 2018]

22. Solve for x: $\tan^{-1}(x - 1) + \tan^{-1} x + \tan^{-1}(x + 1) = \tan^{-1} 3x$.

[CBSE 2014]

23. If $(\tan^{-1})^2 + (\cot^{-1}x)^2 = \frac{5\pi^2}{8}$, find x.

[CBSE 2015]

24. Prove that: $2\tan^{-1}\left(\frac{1}{5}\right) + sec^{-1}\left(\frac{5\sqrt{2}}{7}\right) + 2tan^{-1}\left(\frac{1}{8}\right) = \frac{\pi}{4}$

[CBSE 2014]

25. If $\tan^{-1} x \tan^{-1} y = \frac{\pi}{4}$, $xy < 1$, then write the value of $x + y + xy$.

[CBSE 2014]

26. Write the principal value of $\tan^{-1}\left[sin\left(\frac{-\pi}{2}\right)\right]$

[CBSE 2014]

27. Find the value of $\cot\left[\frac{\pi}{2} - 2cot^{-1}\sqrt{3}\right]$.

[CBSE 2014]

28. Write the value of $\cos^{-1}\left(-\frac{1}{2}\right) + 2sin^{-1}\left(\frac{1}{2}\right)$

[CBSE 2014]

29. Solve for x: $\tan^{-1} x + 2\cot^{-1} x = \frac{2\pi}{3}$

[CBSE 2014]

30. Prove that:

$$\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right) = \frac{x}{2},$$

$$x \in \left(0, \frac{\pi}{4}\right)$$

[CBSE 2014]

31. If $\sin\left(sin^{-1}\frac{1}{5} + cos^{-1}x\right) = 1$ then find the value of x.

[CBSE 2014]

32. Prove that: $\sin^{-1}\left(\frac{8}{17}\right) + sin^{-1}\left(\frac{3}{5}\right) = cos^{-1}\left(\frac{36}{85}\right)$

33. (a) Find the value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + tan^{-1}\left[sin\left(-\frac{\pi}{2}\right)\right]$.

(2024)

OR

(b) Find the domain of the function f(x) = $\sin^{-1}(x^2 - 4)$. Also, find its range.

(2024)

34. Evaluate : $\sec^2\left(\tan^{-1}\frac{1}{2}\right) + cosec^2\left(\cot^{-1}\frac{1}{2}\right)$

(2024)

35. Find the value of k if

$$\sin^{-1}\left[k\tan\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right] = \frac{\pi}{3}.$$

(2024)

Long Qs (4 - 5 marks)

36. Solve $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$.

[CBSE 2019, 15]

37. Find the value of $\sin\left(\cos^{-1}\frac{4}{5} + \tan^{-1}\frac{2}{3}\right)$.

[CBSE 2014]

38. Prove that: $\cos^{-1}\frac{12}{13} + \sin^{-1}\frac{3}{5} = \sin^{-1}\frac{56}{65}$

[CBSE 2019]

39. Solve: $\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}\frac{8}{31}$.

[CBSE 2019, 15]

40. Prove that: $2\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \sin^{-1}\left(\frac{1}{25\sqrt{2}}\right)$

[CBSE 2019]

Class – 12 Mathematics

PYQ Solutions

Chapter – 2

Inverse Trigonometric Functions

1. (a) 1

Explanation:

$$\sin\left[\sin^{-1}\left(\frac{1}{2}\right) + \cos^{-1}\left(\frac{1}{2}\right)\right]$$
$$= \sin\left(\sin^{-1}\frac{1}{2}\right)\cos\left(\cos^{-1}\frac{1}{2}\right) + \sin\left(\sin^{-1}\frac{1}{2}\right)\cos\left(\cos^{-1}\frac{1}{2}\right)$$
$$= \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$$
$$= \frac{1}{4} + \frac{3}{4}$$
$$= 1$$

2. (b) $-\frac{\pi}{10}$

Explanation: $\sin^{-1}\left(\cos\frac{13\pi}{5}\right)$

$=\sin^{-1}\left(\cos\left(2\pi + \frac{3\pi}{5}\right)\right)$

$=\cos^{-1}\left(\cos\frac{3\pi}{5}\right)$

Since, $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$

$$\sin^{-1}\left(\cos\frac{3\pi}{5}\right) = \sin^{-1}\left(\sin\left(\frac{\pi}{2} - \frac{3\pi}{2}\right)\right)$$
$$\sin^{-1}\left(\sin\left(-\frac{\pi}{10}\right)\right)$$
$$= -\frac{\pi}{10}$$

3. (d) $\frac{\pi}{6}$

Explanation:

$\sec^{-1}(2) + \sin^{-1}\left(\frac{1}{2}\right) + \tan^{-1}(-\sqrt{3})$

$= \frac{\pi}{3} + \frac{\pi}{6} - \frac{\pi}{3} = \frac{\pi}{6}$

4. (a) a= 0, b = π

Explanation:

We know that, $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$

and $\frac{-\pi}{2} \leq \sin^{-1} ? \leq \frac{\pi}{2}$,

So, $0 \leq \sin^{-1} x + \frac{\pi}{2} \leq \pi$

$\Rightarrow 0 \leq 2\sin^{-1} x + \cos^{-1} x \leq \pi$

Thus a = 0, b = π

5. (c) $\frac{-\pi}{2} < y < \frac{\pi}{2}$

6. (D) 4

Explanation: Given, $\tan^{-1} x = y$

We know that, the range of $\tan^{-1}$ function is

$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

i.e., $-\frac{\pi}{2} < \tan^{-1} x < \frac{\pi}{2}$

$\Rightarrow -\frac{\pi}{2} < y \frac{\pi}{2}$

7. (b) $\frac{\pi}{2}$

Explanation:

$\cos^{-1} x + \cos^{-1} x = \pi - (\sin^{-1} x + \sin^{-1} x)$

$= \pi - \frac{2\pi}{3} = \frac{\pi}{3}$

8. (b) $-\frac{\pi}{2}$

Explanation: $\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$

$= tan^{-1}\left(tan\frac{\pi}{3}\right) - cot^{-1}\left(- cot^{-1}\frac{\pi}{6}\right)$

$= tan^{-1}\left(tan\frac{\pi}{3}\right) - cot^{-1}\left[cot\left(\pi - \frac{\pi}{6}\right)\right]$

$= \frac{\pi}{3} - \pi + \frac{\pi}{6} = -\frac{\pi}{2}$

9. Both (A) and (R) are false.

Explanation: From the graph of $\cos^{-1} x$ it is clear that, the value of $\cos^{-1} x$ can be $2\pi, \frac{5\pi}{2} \ldots$

Hence, π^2 cannot be the maximum value of $(\cos^{-1} x)^2$

We know that,

Principal value branch of $\cos^{-1} x$ is $[0, \pi]$

10. (c) (A) is true but (R) is false.

Explanation: $\sec^{-1} x$ is defined if $x \leq -1$ or $x \geq 1$. Hence, $\sec^{-1} 2x$ will be defined if $x \leq -\frac{1}{2}$ or $x \geq \frac{1}{2}$

. The range of the function $\sec^{-1} x$ is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$

11. (d) (A) is false, but (R) is true.

Explanation:

The principal value branch of $\sin^{-1} x$ is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

Let x = sin x

$\Rightarrow \theta = \sin^{-1} x$

$\sin^{-1}(\sin \theta) = \sin^{-1} x = \theta$

$\frac{2\pi}{3} \neq \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

$\sin^{-1}(\sin \theta) = \theta$, if $\theta \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

12. (A) Assertion (A) is true, but Reason (R) is false.

13. $\sin^{-1}\left(cos\left(\frac{33\pi}{5}\right)\right)$

$= \sin^{-1}\left(cos\left(6\pi + \frac{3\pi}{5}\right)\right)$

$= \sin^{-1} cos\left(\frac{3\pi}{5}\right)$

$= \sin^{-1} sin\left(\frac{\pi}{2} - \frac{3\pi}{5}\right)$

$= \frac{\pi}{2} - \frac{3\pi}{5}$

$= \frac{\pi}{10}$

14. $-1 \leq (x^2 - 4) \leq 1$

$\Rightarrow 3 \leq x^2 \leq 5$

$\Rightarrow$

$\sqrt{3} \leq |x| \leq 5$

$\Rightarrow x \in [-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$.

So, required domain is $[-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$

15. $\sin^{-1}\left(\sin\frac{3\pi}{4}\right) + \cos^{-1}(\cos\pi) + \tan^{-1}(1)$

$= \sin^{-1}\left(\sin\pi - \frac{\pi}{4}\right) + \cos^{-1}(\cos\pi) + \tan^{-1}(\frac{\pi}{4})$

$= \sin^{-1}\left(\sin\frac{\pi}{4}\right) + \pi + \frac{\pi}{4}$ [$\because \sin(\pi - \theta) = \sin\theta$]

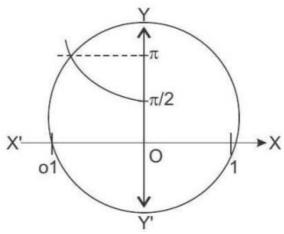
$= \frac{\pi}{4} + \pi + \frac{\pi}{4}$

$= \frac{3\pi}{2}$

16. Given :

$y = \cos^{-1} x$, where $x \in [-1, 0]$

range$\left[\frac{\pi}{2}, \pi\right]$



17. Given, $\sin^{-1}\left[\sin\left(\frac{13\pi}{7}\right)\right]$.

$= \sin^{-1}\left[\sin\left(2\pi - \frac{\pi}{7}\right)\right]$

$= \sin^{-1}\left[-\sin\left(\frac{\pi}{7}\right)\right]$

$= \sin^{-1}\left[\sin\left(\frac{-\pi}{7}\right)\right]$

We Know that, range of $\sin^{-1}\left[\sin\left(\frac{13\pi}{7}\right)\right] = \frac{-\pi}{7}$.

18. Let

$x = \cos\theta$

$\Rightarrow \theta = \cos^{-1}x$

$\therefore$ L.H.S. $= \sin^{-1}\left(2\cos\theta\sqrt{1 - \cos^2\theta}\right)$

$= \sin^{-1}(2\cos\theta\sin\theta)$

$= \sin^{-1}(\sin 2\theta) = 2\theta$

$= 2\cos^{-1} x$ = R.H.S

Hence, proved.

19. Let

$\Rightarrow \theta = \sin^{-1} x$

R.H.S. $= \sin^{-1}(3\sin\theta - 4\sin^3\theta)$

$= \sin^{-1}(\sin 3\theta)$

$\because \sin 3\theta = 3\sin\theta - 4\sin^3\theta$

$= 3\theta = 3\sin^{-1}\theta =$ L.H.S.

Hence, proved.

20. $\tan^{-1}\sqrt{3} - sec^{-1}(-2)$

$= \frac{\pi}{3} - \left(\pi - \frac{\pi}{3}\right)$

$= \frac{\pi}{3} - \pi + \frac{\pi}{3}$

$= \frac{2\pi}{3} - \pi$

$= -\frac{\pi}{3}$

21. R.H.S $= \cos^{-1}(4x^3 - 3x)$

Putting $x = \cos\theta$

$= \cos^{-1}(4\cos\theta^3 - 3\cos\theta)$

$= \cos^{-1}(\cos 3\theta)$

$= 3\theta(\because \cos 3x = 4\cos^3 x - 3\cos x)$

$= 3\theta(\because \cos^{-1}(\cos x) = x)$

Now,

$x = \cos\theta$

$\therefore \cos^{-1}(x) = \theta$

$\therefore 3\theta = 3\cos^{-1}(x)$

$=$ L.H.S.

Hence, proved.

22. $\tan^{-1}(x-1) + \tan^{-1} x + \tan^{-1}(x+1) = \tan^{-1} 3x \tan^{-1}(x-1) + \tan^{-1}(x+1) = \tan^{-1} 3x \tan^{-1} x$

or $tan^{-1}\left(\frac{2x}{2-x^2}\right) = tan^{-1}\left(\frac{2x}{1+3x^2}\right)$

$or\ \frac{2x}{2-x^2} = \frac{2x}{1+3x^2}$

or $2x(1 + 3x^2 - 2 + x^2) = 0$

or $2x(4x^2 - 1) = 0$

or $x = 0, \frac{1}{2}, -\frac{1}{2}$

23. Given, $(\tan^{-1})^2 + (\cot^{-1}x)^2 = \frac{5\pi^2}{8}$

$\Rightarrow (\tan^{-1} x)^2 + \left(\frac{\pi}{2} - tan^{-1} x\right)^2 = \frac{5\pi^2}{8}$

$\Rightarrow 2(\tan^{-1} x)^2 - \pi \tan^{-1} x + \frac{\pi^2}{4} - \frac{5\pi^2}{8} = 0$

$\Rightarrow 2(\tan^{-1} x)^2 - \pi \tan^{-1} x - \frac{3\pi^2}{8} = 0$

$\Rightarrow \tan^{-1} x = \frac{\pi \pm \sqrt{\pi^2 + 3\pi^2}}{4}$

$\Rightarrow \tan^{-1} x = \frac{3\pi}{4}, \frac{-\pi}{4}$

$\therefore x = -1$

24. L.H.S = $2\left[tan^{-1}\left(\frac{1}{5}\right) + 2tan^{-1}\left(\frac{1}{8}\right)\right] + sec^{-1}\left(\frac{5\sqrt{2}}{7}\right)$

$= tan^{-1}\left(\frac{\frac{1}{5}+\frac{1}{8}}{1-\frac{1}{40}}\right) + tan^{-1}\left(\sqrt{\left(5\sqrt{2}\right)^2 - 1}\right)$

$= tan^{-1}\left(\frac{1}{3}\right) + tan^{-1}\left(\frac{1}{7}\right)$

$= tan^{-1}\left(\frac{\frac{2}{3}}{1-\left(\frac{1}{3}\right)^2}\right) + tan^{-1}\left(\frac{1}{7}\right)$

$= tan^{-1}\left(\frac{3}{4}\right) + tan^{-1}\left(\frac{1}{7}\right)$

$= tan^{-1}\frac{\frac{3}{4}+\frac{1}{7}}{1-\frac{3}{28}} = tan^{-1}\left(\frac{25}{25}\right)$

$= tan^{-1}(1)$

$= \frac{\pi}{4}$

25. We have $\tan^{-1}x \tan^{-1} y = \frac{\pi}{4}$

$$tan^{-1}\left(\frac{x+y}{1-xy}\right) = \frac{\pi}{4}$$

$$\frac{x+y}{1-xy} = tan\frac{\pi}{4} = 1$$

$$x + y = 1 - xy$$

$$x + y + xy = 1$$

26. $\tan^{-1}\left[sin\left(\frac{-\pi}{2}\right)\right] = tan^{-1}[-1]$

$= \tan^{-1}\left(tan\frac{\pi}{4}\right) = \frac{\pi}{4}$

$\because \tan^{-1}(tan\theta) = \theta \forall \theta \epsilon \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

27. $\cot\left[\frac{\pi}{2} - 2cot^{-1}\sqrt{3}\right] = \cot\left[\frac{\pi}{2} - 2 \times \frac{\pi}{6}\right]$

$$\left[\because cot^{-1}\sqrt{3} = \frac{\pi}{6}\right]$$

$$= \cot\left[\frac{\pi}{2} - \frac{\pi}{3}\right]$$

$$= \cot\left[\frac{3\pi - 2\pi}{2}\right]$$

$$= \cot\frac{\pi}{6} = \sqrt{3}$$

28. $\cos^{-1}\left(-\frac{1}{2}\right) + 2sin^{-1}\left(\frac{1}{2}\right)$

$$= cps^{-1}\left(cos\frac{2\pi}{3}\right) + 2sin^{-1}\left(\frac{\pi}{6}\right)$$

$$\left[\because cos\frac{2\pi}{3} = -\frac{1}{2} \; and \; sin\frac{\pi}{6} = \frac{1}{2}\right]$$

$$= \frac{2\pi}{3} + 2 \times \frac{\pi}{6}$$

$$= \frac{2\pi}{3} + \frac{\pi}{3} = \frac{3\pi}{3} = \pi$$

29. $\tan^{-1} x + 2\cot^{-1} x = \frac{2\pi}{3}$

$$\tan^{-1} x + 2\left(\frac{\pi}{2}\ \ tan^{-1}x\right) = \frac{2\pi}{3}$$

Or $\left[\because tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}\right]$

$$-tan^{-1} x = -\frac{\pi}{3}$$

$$tan^{-1} x = \frac{\pi}{3}$$

$$x = \tan\frac{\pi}{3}$$

$$= \sqrt{3}$$

30. $\cot^{-1}\left(\frac{\sqrt{1+sin\,x}+\sqrt{1-sin\,x}}{\sqrt{1+sin\,x}-\sqrt{1-sin\,x}}\right)$

$\left[For\ x \in \left(0,\frac{\pi}{4}\right), 1 \pm sin\,x = cos^2\frac{x}{2} + sin^2\frac{x}{2} \pm 2sin\frac{x}{2}cos\frac{x}{2} = \left(cos\frac{x}{2} \pm sin\frac{x}{2}\right)^2\right]$

$= \cot^{-1}\left[\frac{\sqrt{\left(cos\frac{x}{2}+sin\frac{x}{2}\right)^2}+\sqrt{\left(cos\frac{x}{2}-sin\frac{x}{2}\right)^2}}{\sqrt{\left(cos\frac{x}{2}+sin\frac{x}{2}\right)^2-\left(cos\frac{x}{2}-sin\frac{x}{2}\right)^2}}\right]$

$= \cot^{-1}\left[\frac{cos\frac{x}{2}+sin\frac{x}{2}+cos\frac{x}{2}-sin\frac{x}{2}}{cos\frac{x}{2}+sin\frac{x}{2}-cos\frac{x}{2}+sin\frac{x}{2}}\right]$

$= \cot^{-1}\left[\frac{2cos\frac{x}{2}}{2sin\frac{x}{2}}\right]$

$= \cot^{-1}\left[cot\frac{x}{2}\right] = \frac{x}{2}$

Hence, proved.

31. Given

$$\sin\left(sin^{-1}\frac{1}{5} + cos^{-1}x\right) = 1$$

Putting $\sin\frac{\pi}{2} = 1$

$$\sin\left(sin^{-1}\frac{1}{5} + cos^{-1}x\right) = \sin\frac{\pi}{2}$$

$\Rightarrow sin^{-1}\frac{1}{5} + cos^{-1}x = \frac{\pi}{2}$

$$\Rightarrow sin^{-1}\frac{1}{5} = \frac{\pi}{2} - cos^{-1}x$$

$$\Rightarrow sin^{-1}\frac{1}{5} = sin^{-1}x$$

$$\Rightarrow \frac{1}{5} = x$$

Thus, $x = \frac{1}{5}$

32. Let $\sin^{-1}\left(\frac{8}{17}\right) = \alpha\ and\ sin^{-1}\left(\frac{3}{5}\right) = \beta$

$$\Rightarrow \sin\alpha\frac{8}{17}\ and\ sin\,\beta = \frac{3}{5}$$

$$\Rightarrow \cos\alpha = \sqrt{1 - \sin^2\alpha}$$

and $\cos\beta = \sqrt{1 - \sin^2\beta}$

$\Rightarrow \cos\alpha = \sqrt{1 - \frac{64}{289}}$

and $\cos\beta = \sqrt{1 - \frac{9}{25}}$

$\Rightarrow \cos\alpha = \sqrt{\frac{289-64}{289}}$

and $\cos\beta = \sqrt{\frac{25-9}{25}}$

$\Rightarrow \cos\alpha = \sqrt{\frac{225}{289}}$

and $\cos\beta = \sqrt{\frac{16}{25}}$

$\Rightarrow \cos\alpha = \frac{15}{17}$ and $\Rightarrow \cos\beta = \frac{4}{5}$

Now, $\cos(\alpha + \beta) = \cos\alpha . \cos\beta - \sin\alpha . \sin\beta$

$\Rightarrow \cos(\alpha + \beta) = \frac{15}{17} \times \frac{4}{5} - \frac{8}{17} \times \frac{3}{5}$

$\Rightarrow \cos(\alpha + \beta) = \frac{60}{85} - \frac{24}{85}$

$\Rightarrow \cos(\alpha + \beta) = \frac{36}{85}$

$\Rightarrow \alpha + \beta = \cos^{-1}\left(\frac{36}{85}\right)$

[Putting the value of α and β]

33. (a)

1. Evaluate $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$:

$$\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) = -\tan-1\left(\frac{1}{\sqrt{3}}\right)$$

We know that:

$$tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$$

Therefor:

$$tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = -\frac{\pi}{6}$$

2. Evaluate $\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$:

$$\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right) = tan^{-1}(\sqrt{3})$$

We Know that

$$tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$$

3. $Evaluate\ tan^{-1}\left[sin\left(-\frac{\pi}{2}\right)\right]$

$$sin\left(-\frac{\pi}{2}\right) = -1$$

Therefor:

$$tan^{-1}(-1) = -\frac{\pi}{4}$$

4. Sum the evaluated terms:

$$-\frac{\pi}{6}+\frac{\pi}{3}= -\frac{\pi}{4}$$

5. **Combine the fractions:**
   Converting all terms to a common denominator (12):
$$-\frac{2\pi}{12}+\frac{4\pi}{12}-\frac{3\pi}{12}=\frac{-2\pi+4\pi-3\pi}{12} = \frac{-\pi}{12}$$
   Therefore, the value of the given expression is $-\frac{\pi}{12}$ (2024)

OR

(b) To find the domain and range of the function $f(x) = \sin^{-1}(x^2 - 4)$, we need to ensure that the expression inside the inverse sine function lies within its principal range.

For $\sin^{-1}(x^2 - 4)$ to be defined, the argument must lie in the interval $x^2 - 4$ must lie in the interval $[-1, 1]$

Therefore, we need:

$-1 \leq x^2 - 4 \leq 1.$

Solving these inequalities:

1. **First Inequality:**

$x^2 - 4 \geq -1$

$x^2 \geq 3$

$x \leq -3$ or

$x \geq 3$

2. **Second Inequality:**

$x^2 - 4 \leq 1$

$x^2 \leq \sqrt{5}$

$-\sqrt{5} \leq x \leq \sqrt{5}$

Combining both conditions, we get:

$-\sqrt{5} \leq x \leq -\sqrt{3}$ or $\sqrt{3} \leq x \leq \sqrt{5}$

Thus, the domain of the function $f(x) = \sin^{-1}(x^2 - 4)$ is:

$x \in [-\sqrt{5}, -3] \cup [\sqrt{3}, \sqrt{5}]$

To find the range of , we consider the values of $x^2 - 4$ within the interval. $[-1, 1]$

The function $x^2 - 4$ attains its minimum value of -1 when $x = \pm\sqrt{3}$ and its maximum value of when . 1 $x = \pm\sqrt{5}$

Hence, the range of the function $f(x)$ is

$$[sin^{-1}(-1), sin^{-1}(1)]$$
$$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

34.

1. Evaluate $\sec^2\left(tan^{-1}\left(\frac{1}{2}\right)\right)$:

Let $\theta = \tan^{-1}\left(\frac{1}{2}\right)$. Then , $\tan\theta = \frac{1}{2}$.

We Know that

$$\sec^2\theta = 1 + \tan^2\theta$$

So,

$$\sec^2\left(\tan^{-1}\left(\frac{1}{2}\right)\right) = 1 + \left(\frac{1}{2}\right)^2 = 1 + \frac{1}{4} = \frac{5}{4}$$

2. Evaluate cosec$^2\left(cot^{-1}\left(\frac{1}{3}\right)\right)$:

   Let $\emptyset = \cot^{-1}\left(\frac{1}{3}\right)$. The, cot $\emptyset = \frac{1}{3}$

   We Know that:

   cosec^2 $\emptyset$ =1+ $\cot^2\emptyset$

   So,

   cosec$^2\left(cot^{-1}\left(\frac{1}{3}\right)\right) = 1 + \left(\frac{1}{3}\right)^2 = 1 + \frac{1}{9} = \frac{10}{9}$

3. Sum the Results:

$$\sec^2\left(\tan^{-1}\left(\frac{1}{2}\right)\right) + \text{cosec}^2\left(\cot^{-1}\left(\frac{1}{3}\right)\right) = \frac{5}{4} + \frac{10}{9}$$

To add these fractions, find a cooman denominator:

$$\frac{5}{4} = \frac{45}{36}$$

$$\frac{10}{9} = \frac{40}{36}$$

Answer: $\frac{85}{36}$

35. Given the equation:

$$sin^{-1}\left[k\,tan\left(2cos^{-1}\frac{\sqrt{3}}{2}\right)\right] = \frac{\pi}{3}$$

Solutoin:

1. Evaluate $cos^{-1}\frac{\sqrt{3}}{2}$

$$cos^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{6}$$

2. Double the angle:

$$2cos^{-1}\frac{\sqrt{3}}{2} = 2\left(\frac{\pi}{6}\right) = \frac{\pi}{3}$$

3. Evaluate $\tan\left(\frac{\pi}{3}\right)$

   $\tan\left(\frac{\pi}{3}\right) = \sqrt{3}$

4. Substitute back into the equation:

$$sin^{-1}[k\sqrt{3}] = \frac{\pi}{3}$$

5. Solve for *k***:**

We know that:

$$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

So, we set up the equation

$$k\sqrt{3} = \frac{\sqrt{3}}{2}$$

Solving for k;

$$\mathrm{k} = \frac{\sqrt{3}}{2\sqrt{3}} = \frac{1}{2}$$

Therefor, the value of k is:

$$\mathrm{k} = \frac{1}{2}$$

36. Given,

$$\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$$

We Know that,

$$\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$$

$$\tan^{-1}\left(\frac{2x+3y}{1-2x+3y}\right) = \frac{\pi}{4}$$

$$\frac{5x}{1-6x^2} \text{ and } \tan\frac{\pi}{4}$$

$$\frac{5x}{1-6x^2} = 1$$

$$5x = 1-6x^2$$

$$6x^2 + 5x - 1 = 0$$

$$6x^2 + 6x - x - 1 = 0$$

$$6x(x+1) - 1(x+1) = 0$$

$$(6x-1)(x+1) = 0$$

Thus,

$$x = \frac{1}{6} \text{ or } x = -1$$

But for

$$\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$$

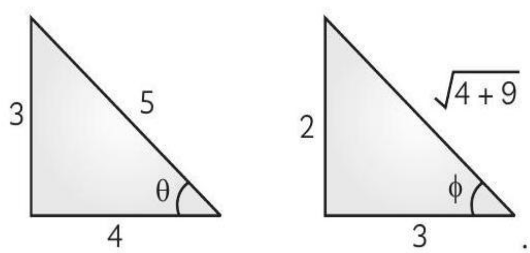
$\tan^{-1}(-2) + \tan^{-1}(-3) = \frac{\pi}{4}$

So, L.H.S. becomes negative but R.H.S. is positive.

Thus, $x = -1$ is not possible.

Hence, $x = \frac{1}{6}$ is the only solution of the given equation.

37. $\sin\left(\cos^{-1}\frac{4}{5} + \tan^{-1}\frac{2}{3}\right)$



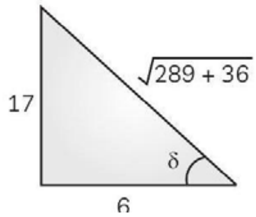
$$\cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{2}{3}\right)$$

$$= \tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{2}{3}\right)$$

$$= \tan^{-1}\left(\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{2}{3}\times\frac{3}{4}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{9+8}{12}}{\frac{12-6}{12}}\right)$$

$$= \tan^{-1}\left(\frac{17}{6}\right)$$



$$\sin\left(\tan^{-1}\frac{17}{6}\right) = \sin\left(\sin^{-1}\frac{17}{\sqrt{325}}\right)$$

38. Let $\alpha = \cos^{-1}\frac{12}{13}$

$$\cos\alpha = \frac{12}{13}$$

We Know that,

$$\sin\alpha = \sqrt{1-\cos^2\alpha}$$

$$= \sqrt{1-\left(\frac{12}{13}\right)^2}$$

$$= \sqrt{\frac{25}{169}} = \frac{5}{13}$$

Let,

$$b = \sin^{-1}\frac{3}{5}$$

$$\sin b = \frac{3}{5}$$

$$\Rightarrow \cos b = \frac{4}{5}$$

We know that
sin (a + b) = sin a cos b + cos a sin b
Putting sin a = $\frac{5}{13}$, cos b = $\frac{12}{13}$ and

$$\sin b = \frac{3}{5}, \cos b = \frac{4}{5}$$

$$\sin(a+b) = \frac{5}{13}\times\frac{4}{5} + \frac{3}{5}\times\frac{4}{5}$$

$$= \frac{20}{65} + \frac{36}{65}$$

$$= \frac{36}{65}$$

Thus,

$$\sin(a+b) = \frac{56}{65}$$

$$a+b = \sin^{-1}\frac{56}{65}$$

$$\Rightarrow \sin^{-1}\frac{12}{13} + \sin^{-1}\frac{3}{5} = \sin^{-1}\frac{56}{65}$$

Thus, L.H.S = R.H.S

Hence, Proved.

39. We have $tan^{-1}(x+1) + tan^{-1}(x-1) = tan^{-1}\frac{8}{31} \Rightarrow tan^{-1}\left[\frac{(x+1)(x-1)}{1-(x^2-1)}\right] = tan^{-1}\frac{8}{31}$

$$\Rightarrow \frac{2x}{2-x^2} = \frac{8}{31}$$

$$\Rightarrow 62x = 16 - 8x^2$$

$$\Rightarrow 8x^2 + 62x + - 16 = 0$$

$$\Rightarrow 4x^2 + 31x + - 8 = 0$$

$$\Rightarrow x = \frac{1}{4} \text{ and } x = -8$$

As x = -8 does not satisfy the equation

Hence, x = $\frac{1}{4}$ isonly solution

40. L.H.S $= 2\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{7}\right)$

$= \tan^{-1}\left(\frac{2.\frac{1}{2}}{1-\left(\frac{1}{2}\right)^2}\right) + \tan^{-1}\left(\frac{1}{7}\right)$

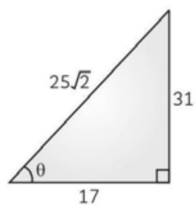
$$= \tan^{-1}\left(\frac{4}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right)$$

$$= \tan^{-1}\left(\frac{\frac{4}{3}+\frac{1}{7}}{1-\left(\frac{4}{3}\right)\left(\frac{1}{7}\right)}\right)$$

$$= \tan^{-1}\left(\frac{\frac{31}{21}}{\frac{17}{2}}\right)$$

$$= \tan^{-1}\left(\frac{31}{17}\right)$$

Let, $\tan^{-1}\left(\frac{31}{17}\right) = \theta$



Then, $\tan\theta = \frac{31}{17}$

$$\therefore \sin\theta = \frac{31}{25\sqrt{2}}$$

$$\Rightarrow \theta \sin^{-1}\left(\frac{31}{25\sqrt{2}}\right)$$

$$\therefore \tan^{-1}\left(\frac{31}{17}\right) = \sin^{-1}\left(\frac{31}{25\sqrt{2}}\right)$$

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